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Locally wrapped D-branes

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Abstract: We find examples of locally wrapped D-branes in string theory. These excitations mimic skyrmions in that they correspond to topological excitations of the scalar fields parametrizing the brane motion in the space transverse to its world-volume. While these brane excitations appear to be point-like, evidence is provided that curvature corrections to the probe action might allow for a delocalization of the wrapping on a scale of the order of the string length, therefore rendering the phenomena non-singular.

Keywords: Solitons Monopoles and Instantons, p-branes, D-branes.

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Contents

1. Introduction

Dirichlet branes play a crucial role in string theory. Along with the fundamental strings, they are the probes with which one must study stringy backgrounds in order to discover the true nature of quantum geometry. It is therefore important to understand the ways in which D-branes can be excited when they are placed in the curved backgrounds generated by string theory objects. In this work, we study a generic manner in which branes can be excited when placed in geometries possessing a compact manifold transverse to their world-volume. The main requirement will be that there exists a non-trivial homotopy mapping for the spatial dimensions of the probe world-volume onto this compact manifold. The excitations we find correspond to the probe world-volume being wrapped, at a single point, onto the transverse manifold. The resulting system consists of two orthogonal branes intersecting at one point but we give evidence that curvature corrections to the probe action might allow for the wrapping to become delocalized on a scale typical of the string length. Locally wrapped branes mimic skyrmions [1, 2] as they are collective excitations of the open string scalar fields parametrizing the D-brane motion in the space transverse to its world-volume.

The paper is organized as follows: In section 2 we give a general description of locally wrapped D-branes based on the Dirac-Born-Infeld (DBI) action. We numerically find an expression for the energy of the excitations. In section 3 we consider a typical α' curvature correction obtained from bosonic string theory and show how it leads to a delocalization of the wrapping. In section 4 we describe both supersymmetric and non-supersymmetric examples in string and M-theory where brane probes can be locally wrapped. We conclude by discussing to some extent the contribution of the α' curvature corrections as well as an extension of our approach to more general backgrounds. Finally, an appendix introduces a scheme allowing us to analytically describe locally wrapped D3-branes approximately, using the Atiyah-Manton ansatz.

2. Locally wrapped D-branes

We refer to a 'locally wrapped brane' (LWB) as a collective excitation of open string degrees of freedom associated with the world-volume of a D-brane. These degrees of freedom are the scalar fields characterizing the motion of the brane in directions that are transverse to its world-volume. The topological brane excitations we are studying are analogous to branes that are locally protruding in directions that are transverse to their world-volume by wrapping a compact manifold. The end result is basically two branes intersecting at right angles. In string and M-theory, one often encounters so-called wrapped D- or M-branes (see, e.g., ref. [3]). These are not the same as the states we are introducing in this paper. Wrapped branes have some or all of their directions wrapped on a compact manifold. Here, we consider a different kind of wrapping mechanism where only a finite region of the world-volume is effectively being wrapped. The world-volume outside of this region is unaffected.

Our approach to studying the LWB's is based on the action of a D-brane probe, the Dirac-Born-Infeld (DBI) action, in some gravitational background of the form[1]

$$ds^2 = G_{\mu\nu}dx^\mu dx^\nu = \frac{1}{H(\rho)^{k_1}}\left[-dt^2 + dx^m dx^m\right] + H(\rho)^{k_2}\left[d\rho^2 + \rho^2 d\Omega^2_{d-p-2}\right], \tag{2.1}$$

$$e^\Phi = \frac{1}{H(\rho)^{k_3}}, \tag{2.2}$$

where $m = 1, \ldots, p$ so that $d = p+q+2$ and the isometry group is $\mathrm{SO}(p,1)\times\mathrm{SO}(d-p-1)$. The metric is here supplemented with a dilaton field, Φ. The form of the metric and the expression for the dilaton are inspired by the fields sourced by supergravity p-branes. The exponents k_1, k_2 and k_3 are taken as arbitrary values for now as this will be useful later on when we enlarge the scope to a more general analysis. We consider placing a Dq-brane probe on the background corresponding to eqs. (2.1) and (2.2). We also use up $(q+1)$ gauge degrees of freedom to align the corresponding $(q+1)$-dimensional world-volume of the probe parallel to the $t - x^m$ coordinates of the background geometry. The brane is then point-like in the transverse space parametrized by the coordinates ρ and Ω_{d-p-2}. As pointed out above, the brane probe motion in the background is, to a good approximation, governed by the DBI action [4, 5, 6],

$$S_{\mathrm{DBI}} = -T_q \int d^{q+1}\sigma\, e^{-\Phi}\sqrt{-|P[G+B]_{ab} + 2\pi\alpha' F_{ab}|}, \tag{2.3}$$

[1]To be consistent we should include a flux associated with the $(p+1)$-form field but it does not affect the soliton configurations we are considering here.

where $a, b = 0, \ldots, q$. $G_{\mu\nu}$, $B_{\mu\nu}$ (μ, $\nu = 0, 1, \ldots, d-1$) and Φ are the background fields coming from the NS-NS sector of the string theory. They are respectively the graviton, the Kalb-Ramond and the dilaton fields. F_{ab} is a U(1) gauge field associated with open string world-volume excitations and $P[\ldots]$ represents the pull-back operation. The probe action contains the coupling terms between the induced background fields $G_{\mu\nu}$, $B_{\mu\nu}$, Φ to the open string world-volume excitations. As well as the U(1) field, the brane excitations include scalar fields parametrizing their motion in directions transverse to the world-volume. The DBI action must be supplemented with the Chern-Simons action including couplings of the world-volume to fields from the R-R sector. This part of the action is not playing a role in the upcoming analysis (see section 4.1 for details). The action eq. (2.3) is valid to all orders in α' for couplings involving F_{ab} when the derivatives of field strenght vanish.

In this work we look at the case where both B_{ab} and F_{ab} are zero. Studying the effect of these fields might lead to interesting results and represents a task in itself so we leave that for future investigations. The simplified version of the DBI action is then

$$S_{\mathrm{DBI}} = -T_q \int d^{q+1}\sigma \; e^{-\Phi} \sqrt{-|P[G]_{ab}|}. \tag{2.4}$$

In section 3 we consider how α' curvature corrections to this action might affect the excitations we now present. Let us consider, for now, placing a Dq-brane probe at fixed ρ in a background of the form (2.1) with dilaton field given by eq. (2.2).[2] The expression for the induced metric on the probe is

$$P[G]_{ab} = G_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^a} \frac{\partial X^\nu}{\partial \sigma^b} = G_{ab} + G_{i(a}\partial_{b)} X^i + G_{ij} \partial_a X^i \partial_b X^j \,, \tag{2.5}$$

where the $(d-p-2)$ scalar fields X^i ($i = p+2, \ldots, d-1$) are parametrizing the brane motion on the compact space $\mathbf{S}^{d-p-2}$ associated with the factor $d\Omega^2_{d-p-2}$ in the metric eq. (2.1). Since the probe is assumed to be fixed at a finite ρ, we take $\partial_a X^{p+1} = \partial_a \rho = 0$.[3] For the cases we are interested in, $G_{\mu\nu}$ has no off-diagonal terms so the pull-back of the gravitational field on the world-volume is simply

$$P[G]_{ab} = G_{ab} + G_{ij} \partial_a X^i \partial_b X^j \,. \tag{2.6}$$

It is useful for our purposes to write down the background metric eq. (2.1) in the form

$$ds^2 = \frac{1}{H(\rho)^{k_1}} \left[-dt^2 + dr^2 + r^2 d\Omega^2_{p-1}\right] + H(\rho)^{k_2} \left[d\rho^2 + \rho^2 \left(d\xi^2 + \sin^2\xi \, d\overline{\Omega}^2_{d-p-3}\right)\right], \tag{2.7}$$

where $0 \leq \xi \leq \pi$. The first term in this metric is simply a conformally flat $(p+1)$-dimensional factor written in spherical coordinates while the transverse part can be regarded as a line (parametrized by ρ) times a sphere $\mathbf{S}^{d-p-2}$ with its volume $V_{\mathbf{S}^{d-p-2}}$ computed with the rescaled radius

$$R = \rho H(\rho)^{\frac{k_2}{2}} \,. \tag{2.8}$$

[2] This assumption is, at this point, unrealistic since only in certain cases will there be an exact cancellation between the forces exerted on the D-brane probe by the graviton, dilaton and R-R form field. We will consider such stable systems in section 4.

[3] In section 5 we comment on the implications that relaxing this condition will have on the locally wrapped branes.

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The q-dimensional space-like part of the probe world-volume can be viewed as being composed of the points on $\mathbf{R}^q$ plus a point corresponding to the 'boundary' at spatial infinity. It is then possible to compactify this infinite hyperplane plus a point on a sphere $\mathbf{S}^q$ of infinite radius. This is a correct idealization of a q-dimensional plane as the curvature of an infinitely large sphere vanishes exactly. The brane excitations we introduce consist of static, spherically symmetric mappings of this infinitely large hyperplane plus a point onto the transverse manifold which, when considering the background in eq. (2.7), is the sphere $\mathbf{S}^{d-p-2}$ with radius given by eq. (2.8). These solutions are classified according to the homotopy class

$$\Pi_q(\mathbf{S}^{d-p-2}) \tag{2.9}$$

to which they belong. Let us recall the following identities which we are going to use repeatedly in the rest of the paper:

$$\Pi_q(\mathbf{S}^{d-p-2}) = \begin{cases} 0 & \text{for } d-p-2 > q \\ \mathbf{Z} & \text{for } d-p-2 = q\,. \end{cases} \tag{2.10}$$

We consider implementing a mapping characterized by the homotopy class eq. (2.9) when $q = d-p-2$. This is accomplished by using an ansatz which associates every point on the q-dimensional hyperplane to point(s) on the compact manifold $\mathbf{S}^q$.[4] More specifically, if every point on the hyperplane is associated with ω points on the compact transverse space, then the solution is in the class: $\Pi_p(\mathbf{S}^p) = \omega$. The latter is the winding number, i.e., the number of times the brane wraps the compact manifold. The metric eq. (2.7) can then be conveniently written in the form

$$ds^2 = \frac{1}{H(\rho)^{k_1}}\left[-dt^2 + dr^2 + r^2 d\Omega_{q-1}^2 + dx^I dx^I\right] + H(\rho)^{k_2}\left[d\rho^2 + \rho^2\left(d\xi^2 + \sin^2\xi\, d\overline{\Omega}_{q-1}^2\right)\right], \tag{2.11}$$

where x^I $(I = q+1, \ldots, p)$ represents directions that are transverse to the world-volume of the probe. For unit winding number, we use the time-independent spherically symmetric ansatz

$$\Omega_{q-1} = \overline{\Omega}_{q-1}\,, \tag{2.12}$$

$$\xi = F(r)\,, \tag{2.13}$$

where $F(r)$ will be called the profile angle for reasons that will become obvious below. It takes values from 0 to π while the coordinate r varies from $+\infty$ to 0 on the world-volume. We supplement the ansatz associated with eqs. (2.12) and (2.13) with the requirement that the probe be fixed at $\rho = \text{const}$.[5] The resulting configuration is associated with a conserved topological charge, i.e., the winding number ω. The spherically symmetric ansatz maps

[4]We consider in detail only simple mappings with $q = d-p-2$ since these are the only ones for which we could find a simple ansatz.

[5]This, stricly speaking, can only be valid when the system preserves some supersymmetries. For a BPS configuration, we take the point of view that the topological excitation on the probe, being only a small perturbation, will not modify the boundary conditions on the D-brane world-volume so as to destabilize the system.

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the 'boundary' of the q-dimensional hyperplane (the region $r \to \infty$) to a single point i.e., the north pole of the compact $\mathbf{S}^q$ (the point associated with $\xi = 0$). The remaining points on the plane, i.e., $\mathbf{R}^q - \{\infty\}$, are mapped to points on a compact q-sphere minus its north pole.

In order to understand the physical significance of the solutions associated with such mappings, let us consider a toy example. Suppose that $F(r)$ is a monotonically decreasing function of r with boundary values $F(0) = \pi$ and $F(\infty) = 0$. Above some critical radius r_c, the profile angle may be considered as zero for all practical means. Accordingly, the map associated with (2.12) and (2.13) then implies that all points for which $r > r_c$ are mapped to a single point on the compact manifold $\mathbf{S}^q$, i.e., the north pole. The points for which $r \le r_c$ are then in one-to-one correspondence with the points on $\mathbf{S}^q$ minus the north pole. In other words, given the radial quantity r_c is small (for example, it could be of the order of the string length, $r_c \sim l_s$) the map associated with $\Pi_q(\mathbf{S}^q) = 1$ corresponds to a D-brane wrapping a co-cycle $\mathbf{S}^q$ but only locally, i.e., in the region $r \le r_c$. For $r > r_c$ the brane probe is point-like on the transverse $\mathbf{S}^q$ and it therefore breaks invariance under the $\mathrm{SO}(q+1)$ group associated with the compact target space, a symmetry which is restored in the region $r \le r_c$.

We now consider LWB's in a more concrete manner. First, let us evaluate the gravitational field induced on a Dq-brane probe in the background geometry (2.7),

$$P[G]_{00} = -\frac{1}{H(\rho)^{k_1}}\,, \tag{2.14}$$

$$P[G]_{rr} = \frac{1}{H(\rho)^{k_1}} + \rho^2 H(\rho)^{k_2} F'^2\,, \tag{2.15}$$

where a 'prime' denotes a derivative with respect to r. The components associated with the $(q-1)$ variables $(\theta_1, \ldots, \theta_{q-2}, \phi)$ on the world-volume of the D-brane are

$$P[G]_{\theta_1\theta_1} \cdots P[G]_{\theta_{q-2}\theta_{q-2}} P[G]_{\phi\phi} = \sin^{2q-4}\theta_1 \cdots \sin^2\theta_{q-2} \left(\frac{r^2}{H(\rho)^{k_1}} + \rho^2 H(\rho)^{k_2}\sin^2 F\right)^{p-1}, \tag{2.16}$$

where the angular variables are those defined on a q-dimensional plane with ϕ an azimuthal angle. The solution is time-independent so the energy functional is simply $E = -\int d^q\sigma \mathcal{L}_{\mathrm{DBI}}$. The equations of motion associated with the profile angle $F(r)$ are obtained by varying the time independent action with Dirichlet boundary conditions $F(0) = \pi$ and $F(\infty) = 0$,

$$\begin{aligned} 0 = &-(q-1)\sin F\cos F - (q-1)\rho^2 H(\rho)^{k_1+k_2} F'^2 \sin F \cos F + (q-1) r F' + \\ &+(q-1)\rho^2 H(\rho)^{k_1+k_2} r F'^3 + F''\left(r^2 + \rho^2 H(\rho)^{k_1+k_2}\sin^2 F\right). \end{aligned} \tag{2.17}$$

At this point, it should become obvious that the topological nature of the solution, the spherically symmetric ansatz for the $\omega = 1$ solution and, finally, this last non-linear equation, are all reminiscent of the Skyrme model [1, 2, 7] where one can also define a similar profile angle $F(r)$. Yet there are significant differences: The Skyrme model leads to three-dimensional solitons interpreted as baryons (two-dimensional for the so-called baby-skyrmions) while our solitons extend in q spatial dimensions. Skyrme introduced an ad-hoc interaction term in his model whereas the interactions here are gravitational in nature. The non-polynomial nature of the lagrangian leads to a more complex equation for $F(r)$. Moreover, the complexity of eq. (2.17) could be an obstacle to finding topological solitonic solutions and indeed there is little hope of finding an exact analytical solution even if the mapping with $\Pi_q(\mathbf{S}^q) = \mathbf{Z}$ implies the existence of such solutions. We must therefore resort to numerical techniques to find solutions for different values of q. Fortunately, it turns out that the techniques used to solve the profile angle for the Skyrme model are appropriate in our case as well [7]. First, because of the singular behavior of some terms in the equation at $r = 0$, one introduces an analytical solution valid for small r, say $r < r_0$, with the form $F(r) = \pi - ar$ where $a = F'(0)$. The rest of the solution is integrated numerically ajusting the parameter a so that $F(\infty) = 0$. This procedure reveals that the energy is sensitive to the choice of r_0 and turns out to be minimum in the limit $r_0 \to 0$, in which case the contribution comes entirely from the region $r < r_0$. This corresponds to a point-like configuration or an exact profile angle which is the step function

$$F(r) \sim \frac{\pi}{2}\,(1 + \theta(r)) \qquad r \geq 0, \tag{2.18}$$

where $\theta(x) = 1$ for $x \leq 0$ and $\theta(x) = -1$ for $x > 0$. Consequently, the minimal energy solution corresponds to a homotopy mapping of the points in the q-dimensional plane onto the north pole of the compact sphere $\mathbf{S}^q$ and of the origin of $\mathbf{R}^p$ onto the other points of the target sphere. It would be wrong to conclude that this configuration corresponds to no wrapping at all. In fact, the energy of the configuration is

$$E = T_q H(\rho)^{k_3 - \frac{pk_1}{2}} V_{\mathbf{S}^q}(R)\,, \tag{2.19}$$

where the volume $V_{\mathbf{S}^q}(R)$ is calculated using the radius R defined in eq. (2.8). One should regard this configuration as a local wrapping corresponding to taking the continuous limit $r_c \to 0$ in the toy example described earlier. The finiteness of the energy in this limit is attributed to the fact that it is impossible to change the topological sector of an excitation by any continuous process. In the next section we present evidence that the α' curvature corrections to the DBI action might allow the wrapping to expand to a size $r_c \sim \sqrt{\alpha'} = l_s$.

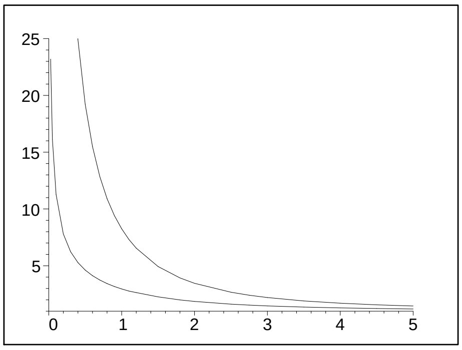

Figure 1: The ratio $E/V_{\mathbf{S}^p}(\rho)$ of the configuration for T_p=1 and $H(\rho)$=1 as a function of R. The lower (upper) curve corresponds to the case $p = 3$ ($p = 4$).

3. Evidence for delocalization of the wrapping

The DBI action with the U(1) gauge field turned off is the first term of an expansion in the parameter α'. In other words, the DBI action in eq. (2.4) constitutes the $\alpha' = 0$ limit of that expansion, i.e., the field theory limit. This raises the question of how LWB's are affected when α' corrections are switched on. In a sense, it is natural that the $\alpha' = 0$ limit leads to a singular phenomena since the excitations are then derived from a theory where the fundamental length scale has been set to zero. Saying that it is necessary to include the α' corrections is a statement that the curvatures involved in the problem (in our case the curvature of the transverse sphere), are large compared to $1/\alpha'$. For now, we take the point of view that the curvature is large but still small enough that only one extra term needs be considered in the α' expansion. This assumption, and other aspects of the curvature corrections, are discussed in section 5, but for now is motivated by a rather practical reason: only one term in the expansion has been calculated, to our knowledge. In bosonic string theory, the $\mathcal{O}(\alpha')$ correction has been found to be [8] (for totally geodesic branes),

$$-\alpha' T_q \int d^{q+1}\sigma \sqrt{-|P[G]_{ab}|}\mathcal{R}\,, \tag{3.1}$$

where $\mathcal{R}$ is the Ricci scalar evaluated with the induced metric $P[G]_{ab}$. For the D-branes in superstring theory, the $\mathcal{O}(\alpha')$ correction vanishes identically but the $\mathcal{O}(\alpha'^2)$ correction is a sum of terms involving quadratic products of the Riemann and Ricci tensors as well as the Ricci scalar [9, 10, 11, 12].

As an example, we consider adding the term (3.1) to the DBI action eq. (2.4).[6] Once again, we use the spherically symmetric ansatz corresponding to (2.12) and (2.13) and minimize the resulting energy functional which leads to a differential equation for the profile $F(r)$. The new terms that need to be added to eq. (2.17), although easily derived, are numerous and complicated so we do not write them here. Integrating the differential equation using the procedure described above, it becomes clear that the profile is not localized at a point anymore but rather is a monotonically decreasing function of r. In other words, a delocalization of the wrapping has been induced by the $\mathcal{O}(\alpha')$ correction. In figure 1, we illustrate how the energy of the configuration, shown here in terms of the ratio $E/V_{\mathbf{S}^q}(R)$, depends on ρ for a simple non-physical case where $T_q = 1$ and $H(\rho) \equiv 1$ so that the rescaled radius $R = \rho$ here. The lower (upper) curve corresponds to the case $p = 3$ ($p = 4$). In both cases we find that for large values of the target space volume (which corresponds to the $\mathcal{O}(\alpha')$ correction becoming smaller and smaller), the ratio $E/V_{\mathbf{S}^p}(R)$ approaches 1 which corresponds to the result in eq. (2.19). On the other hand, a close analysis reveals that the energy E tends to zero as ρ reaches zero. Delocalization of the configurations becomes evident when one examines how the energy is distributed in space. For that purpose we introduce the "size" of the configuration to be defined, somewhat arbitrarily, as the radius r_s for which the energy density reaches 1% of its maximum value. The results are presented in figure 2 where the mapping becomes

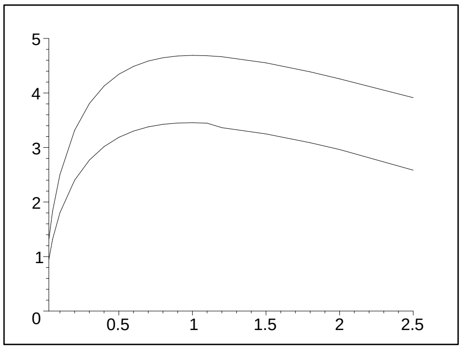

Figure 2: The size r_s of the configuration as a function of ρ for $p = 3$ (lower curve) and $p = 4$. As ρ gets larger the size decreases to a point-like object. On the other hand, the size of the configuration reaches a maximum near $\rho = 1$ in both cases.

[6] Our attempts to include the $\mathcal{O}(\alpha'^2)$ as well have failed due to the highly complicated nature of the corresponding differential equation.

increasingly local for large values of the target space volume. However, the size of the configuration r_s reaches a maximum near $\rho = 1$ in both cases. Finally, for small values of the target space volume where the $\mathcal{O}(\alpha')$ corrections are increasingly large the configuration tends to be point-like again. Of course this last observation is only valid when higher order corrections in α' are not taken into account and clearly these are not negligible in a regime where $V_{\mathbf{S}^q}$ is small.

Although we have gone through the process of solving numerically the profile differential equation, one may, in certain cases, look at the problem using a more intuitive approach. The reader can find in the appendix an example illustrating the delocalization effect by way of using a specific ansatz for the profile $F(r)$ and variational approach. The results are of course not exact but a similar behavior to what we have found numerically is observed for $p = 3$.

4. Applications in string and M-theory

We present examples in string and M-theory where the formalism introduced in section 2 is applicable. We mostly consider brane probes interacting with fields that are sourced by supergravity solutions descending from either string or M-theory. Firstly, we present type-IIA,B superstring theory examples characterized by non-trivial homotopy mappings of the form $\Pi_q(\mathbf{S}^p)$. We then consider applications of the LWB formalism in M-theory, in critical bosonic string theory and, finally, in the speculative bosonic M-theory proposed in ref. [13]. The examples fall in two categories: (1) those with $q = p$ to which the mathematical analysis of section 2 applies directly and (2) those for which $q \neq p$ but that are nevertheless associated with a non-trivial homotopy mapping.

4.1 Superstring theory examples

The low energy limit of the type-IIA,B superstring theories are the ten-dimensional type-IIA,B supergravity theories. The type-IIA theory contains Dp-branes with p even while the type-IIB theory contains branes associated with p odd. The supergravity (low energy) manifestation of these objects, the p-branes, are characterized by an $\mathrm{SO}(1,p) \times \mathrm{SO}(9-p)$ symmetry which is manifest from the gravitational field they generate [14],[7]

$$ds^2 = \frac{1}{H_p(\rho)^{1/2}} \left[-dt^2 + dx^m dx^m\right] + H_p(\rho)^{1/2} \left[d\rho^2 + \rho^2 d\Omega_{8-p}^2\right], \tag{4.1}$$

$$e^\Phi = H_p(\rho)^{\frac{1}{4}(3-p)}. \tag{4.2}$$

These fields correspond to the general solution associated with eqs. (2.1) and (2.2) where $k_1 = k_2 = 1/2$ and $k_3 = (p-3)/4$. Like their microscopic realization, the p-branes are natural sources for the R-R form field [16]

$$C_{01\ldots p}(\rho) = \frac{1}{g_s}\left[1 - \frac{1}{H_p(\rho)}\right]. \tag{4.3}$$

[7] The notation used is that of ref. [15].

The corresponding Chern-Simons term does not affect the physics of a LWB which involves couplings of background fields to the world-volume scalars X^i.[8] $H_p(\rho)$ is a harmonic function in the space transverse to the world-volume of the p-brane,

$$H_p(\rho) = 1 + \frac{c_p g_s N l_s^{7-p}}{\rho^{7-p}}\,, \tag{4.4}$$

where c_p is a dimensionless constant [15], g_s is the string coupling and N is the flux of the R-R field (4.3) through the $\mathbf{S}^{8-p}$ factor of the metric. Strictly speaking, the field expressions (4.1), (4.2) and (4.3) are valid only for $p < 7$. As it stands, we will not need the expressions corresponding to $p \geq 7$ in what follows.

Our approach consists in probing the p-brane geometries with fundamental Dq-branes in such a way as to determine for which values of q a LWB excitation can exist. There are three requirements:

(1) $q \leq p$: because only then can the world-volume of the probe be parallel to the world-volume of the p-brane. It is crucial that the D-brane be orthogonal to the $\mathbf{S}^{8-p}$ factor.

(2) $q \geq 8-p$: otherwise it is guarenteed that the mapping $\Pi_q(\mathbf{S}^{8-p})$ is trivial.

(3) If p is odd (even) then q must be odd (even) since D-branes of opposite parity cannot exist in the same theory.

Leaving aside for now issues of stability, there are ten configurations, in either type-IIA or type-IIB string theory, which satisfy these three criteria. To enumerate those cases, we introduce the notation Dq/p ($\Pi_q(\mathbf{S}^{8-p})$) which refers to a system composed of a Dq-brane probe in the supergravity background generated by a p-brane. The corresponding homotopy mapping $\Pi_q(\mathbf{S}^{8-p})$ is assumed to be non-trivial. The potential superstring theory examples are

Type-IIA	Type-IIB
D2/6($\Pi_2(\mathbf{S}^2)$), D4/4($\Pi_4(\mathbf{S}^4)$),	D1/7($\Pi_1(\mathbf{S}^1)$), D3/5($\Pi_3(\mathbf{S}^3)$),
D4/6($\Pi_4(\mathbf{S}^2)$), D6/6($\Pi_6(\mathbf{S}^2)$),	D3/7($\Pi_3(\mathbf{S}^1)$), D5/5($\Pi_5(\mathbf{S}^3)$),
	D5/7($\Pi_5(\mathbf{S}^1)$), D7/7($\Pi_7(\mathbf{S}^1)$).

Recall that the setup we are considering is a Dq-brane probe placed at a fixed distance from a p-brane. Before considering LWB excitations on the probe, it would seem appropriate to ask whether the system is initially (before the excitation is made to appear) stable or not. Of course, not all of the systems mentionned in the list above are stable. This can be verified straightforwardly by calculating whether the force (due to graviton, dilaton and R-R field exchange) exerted by the p-brane on the probe vanishes or not. It turns out that

[8] There are two instances where the Chern-Simons couplings might be of relevence: (1) when the R-R field depends on the world-volume coordinates. Then, a Taylor expansion in the Chern-Simons action will result in new couplings with the world-volume scalars, (2) in the non-abelian case, couplings of the matrix-valued scalars might be induced by the presence of a higher form field [17].

co-dimension 0 (mod 4) systems are stable (BPS[9]), co-dimension 2 systems are attractive and co-dimension 6 objects are repulsive. Consequently, the only potentially stable systems in the list are

$$\begin{array}{lll} \mathrm{D}2/6(\Pi_2(\mathbf{S}^2))\,, & \mathrm{D}4/4(\Pi_4(\mathbf{S}^4))\,, & \mathrm{D}6/6(\Pi_6(\mathbf{S}^2))\,, \\ \mathrm{D}3/7(\Pi_3(\mathbf{S}^1))\,, & \mathrm{D}5/5(\Pi_5(\mathbf{S}^3))\,, & \mathrm{D}7/7(\Pi_7(\mathbf{S}^1))\,. \end{array} \tag{4.5}$$

The other systems are either attractive, causing the probe to collide with the p-brane, or repulsive. In section 2 we have described a technique allowing one to calculate the energy of the configurations for which $q = 8 - p$. Among the stable configurations, those satisfying this equality are the following type-IIA configurations,

$$\begin{array}{ll} \mathrm{D}2/6(\Pi_2(\mathbf{S}^2)) & \text{with } E = T_2 H_6(R)^{-\frac{3}{4}} V_{\mathbf{S}^2}(R)\,, \\ \mathrm{D}4/4(\Pi_4(\mathbf{S}^4)) & \text{with } E = T_4 H_4(R)^{-\frac{3}{4}} V_{\mathbf{S}^4}(R)\,, \end{array} \tag{4.6}$$

where $R = \rho H_p^{\frac{1}{4}}$ with $p = 6$ and $p = 4$ respectively. There are also configurations among the unstable ones which are such that $q = 8 - p$. Those correspond to the type-IIB systems

$$\begin{array}{ll} \mathrm{D}1/7(\Pi_1(\mathbf{S}^1)) & \text{with } E = T_1 H_7(R)^{-\frac{3}{4}} V_{\mathbf{S}^1}(R)\,, \\ \mathrm{D}3/5(\Pi_3(\mathbf{S}^3)) & \text{with } E = T_3 H_5(R)^{-\frac{3}{4}} V_{\mathbf{S}^3}(R)\,, \end{array} \tag{4.7}$$

which are respectively repulsive and attractive. One can verify that the energy the LWB excitations considered is always a monotonically increasing function of ρ.

4.2 Example involving a M-brane probe

The fundamental branes of M-theory are M2- and M5-branes. The low energy action of M-theory corresponds to eleven-dimensional supergravity with a spectrum comprising of the graviton and a fundamental three-form field. The M2-brane is electrically charged with respect to the three-form field while the M5-brane is a magnetic source for it. The geometry generated by a distribution of N parallel M2-branes is associated with the metric,

$$ds_{11}^2 = \frac{1}{H_{M2}(\rho)^{2/3}} \left[-dt^2 + dx^i dx^i\right] + H_{M2}(\rho)^{1/3} \left[d\rho^2 + \rho^2 d\Omega_7^2\right], \tag{4.8}$$

which has a manifest $\mathrm{SO}(1,2) \times \mathrm{SO}(8)$ symmetry. The harmonic function is

$$H_{M2}(\rho) = 1 + \frac{\rho_-^6}{\rho^6}\,, \tag{4.9}$$

where $\rho_-^6 \sim N$. We consider probing this 'macroscopic' geometry with fundamental M2- and M5-branes. We are interested in finding whether a LWB can be excited on the probes. Having a locally wrapped M5-brane probe is excluded as it does not even satisfy the

[9] More precisely, BPS means that the brane configuration preserves 1/2 of the bulk supersymmetry. The stability of the system can be traced to that fact. Conversely, the unstable systems we are refering to do not preserve any supersymmetries.

criterion (1) introduced in section 4.1. Locally wrapping a M2-brane probe is excluded as well but in this case because such a configuration would violate criterion (2) since $\Pi_2(S^7)$ is trivial.

We now consider the metric associated with a distribution of N parallel M5-branes,

$$ds_{11}^2 = \frac{1}{H_{M5}(\rho)^{1/3}}\left[-dt^2 + dx^i dx^i\right] + H_{M5}(\rho)^{2/3}\left[d\rho^2 + \rho^2 d\Omega_4^2\right], \tag{4.10}$$

which has a manifest $\mathrm{SO}(1,5)\times\mathrm{SO}(5)$ symmetry. The harmonic function is

$$H_{M5}(\rho) = 1 + \frac{\rho_-^3}{\rho^3}. \tag{4.11}$$

In this case, a M2-brane probe cannot be locally wrapped because it violates criterion (2) since $\Pi_2(S^4)$ is again trivial. A M5-brane probe in the background geometry (4.10) can be locally wrapped since $\Pi_5(S^4)$ is non-trivial. In the absence of a convenient ansatz to study the dynamical behavior of the resulting configuration no conclusion can be drawn, though there is no reason to believe the system would behave differently than the type-IIA D4/4 configuration for example. The M5/5 system has the interesting feature of being BPS. Note that if a delocalization process occurs due to higher order curvature corrections, it is in this case expected to be of the order of the eleven-dimensional Planck length, l_P, which is the fundamental scale in the low energy limit of M-theory.

4.3 Locally wrapped bosonic branes

We now comment on the possibility of occurence of LWB excitations in critical bosonic string theory and in the speculative bosonic M-theory proposed in ref. [13].

4.3.1 D-branes in critical bosonic string theory

The D-branes of bosonic string theory are plagued with an open string tachyon making them unstable [18, 19]. While this fact may be a blessing in disguise [20, 21], it is not pleasant for our immediate purposes. In fact, the setup we would like to study is a bosonic Dq-brane placed in a background containing a geometrical factor $\mathbf{S}^p$ in such a way that $\Pi_q(\mathbf{S}^p)$ is non-trivial.[10] The D-brane probe action will include couplings of the open string fields with the world-volume closed string tachyon. This clearly leads to an instability unless the tachyon is assumed to have been taken care of by some unknown mechanism. There is also the problem of the closed string tachyon which is, in a sense, more pressing as it appears to make the whole theory, not just the D-branes, unstable.

Ignoring the tachyon problems, one might imagine inserting a Dq-brane probe in, for example, the background $\mathrm{AdS}_{p+2}\times\mathbf{S}^p\times\mathbf{S}^{24-2p}$ with $d=p+q=26$. These spacetimes have been shown to be stable under small field perturbations (without any supersymmetry) for $q>8$ [22, 23]. The probe action in this case has the same form as the one used in superstring theory, i.e., eq. (2.3) but this time there is no Chern-Simons term. We assume that the

[10] Of course, one would ideally require such a background to be generated by a configuration constructed out of objects coming from bosonic string theory. We will be content here in probing an arbitrary yet stable background.

world-volume of the probe is parallel to $q+1$ directions of the AdS factor. Consequently, the brane will be allowed to be locally wrapped if

- $q \leq p$,
- $\Pi_q(\mathbf{S}^p) \neq 0$ and/or $\Pi_q(\mathbf{S}^{24-p}) \neq 0$.

There is, *a priori*, no reason for the probe not to wrap both spherical co-cycles if the two homotopy mappings are non-trivial. We have given evidence in section 3 that the LWB excitations in bosonic string theory might become delocalized when the α' curvature corrections to the probe action are included.

4.3.2 Branes in bosonic M-theory

We now present an example of a LWB in the context of the speculative 27-dimensional bosonic M-theory proposed by Horowitz and Susskind [13].[11] They proposed that closed bosonic string theory is actually a compactification of a 27-dimensional theory on the orbifold $S^1/\mathbf{Z}_2$ with no extra degrees of freedom living at the fixed points. The authors of ref. [13] have argued that the open string tachyon instability may be removed in the strong coupling limit, allowing them to study branes in this theory. The proposed low energy action for bosonic M-theory is

$$S = \int d^{27}x\sqrt{-g}\left[R - \frac{1}{48}F_{\mu\nu\rho\lambda}F^{\mu\nu\rho\lambda}\right], \tag{4.12}$$

where $F = dC$. The fundamental fields are the graviton and the supergravity three-form which are sourced by M2- and M21-branes. These objects are the extremal limits of black brane solutions [25]. The fields associated with the electrically charged bosonic M2-brane are

$$ds^2 = \frac{1}{H_{M2}(\rho)^{2/3}}\left[-dt^2 + dx^m dx^m\right] + H_{M2}(\rho)^{1/11}\left[d\rho^2 + \rho^2 d\Omega_{23}\right], \tag{4.13}$$

where

$$H_{M2}(\rho) = 1 + \left(\frac{\rho_-}{\rho}\right)^{22}, \tag{4.14}$$

and $\rho_-^{22} \sim N$. The four-form under which the M2-brane is electrically charged is

$${}^*F = N l_P^{22}\epsilon_{23}\,, \tag{4.15}$$

where N is the number of fundamental branes forming the 'macroscopic' configuration, l_P is the 27-dimensional Planck length, and ϵ_{23} is the volume form on a unit $\mathbf{S}^{23}$. The metric associated with a distribution of N M21-branes is

$$ds^2 = \frac{1}{H_{M21}(\rho)^{1/11}}\left[-dt^2 + dx^m dx^m\right] + H_{M21}(\rho)^{2/3}\left[d\rho^2 + \rho^2 d\Omega_4\right], \tag{4.16}$$

[11] See however ref. [24] where it is argued, based on a coset symmetry analysis, that if a 27-dimensional bosonic M-theory exists it does not have a simple local low energy effective action.

where

$$H_{M21}(\rho) = 1 + \left(\frac{\rho_-}{\rho}\right)^3 , \tag{4.17}$$

and $\rho_-^3 \sim N$. The four-form under which the M5-brane is magnetically charged is

$$^*F = N l_P^3 \epsilon_4 \, . \tag{4.18}$$

The extremal branes are completely stable quantum mechanically and there is no force between pairs of extremal branes of the same dimensionality in complete analogy with the branes in superstring and M-theory.

We now consider whether M2- and M21-brane probes can be locally wrapped when placed in the backgrounds corresponding to eqs. (4.13) and (4.16). Following the analysis of the previous sections, we see that only the M21-brane in the background of a 21-brane supergravity solution can be locally wrapped. In fact, the M21-brane can wrap the $\mathbf{S}^4$ since $\Pi_{21}(\mathbf{S}^4)$ is a non-trivial homotopy mapping. Moreover, the system is stable because the attraction due to graviton exchange is exactly cancelled by the repulsion induced by the exchange of fundamental three-form field quanta. This example is in complete analogy with the M5/5 configuration we found in M-theory.

5. Discussion

In this work we have considered D- and M-branes locally wrapping factors with spherical symmetry in geometries that are generated by distributions of other branes. The examples we solved for in detail involve simple homotopy mappings of the form $\Pi_q(\mathbf{S}^q) = \mathbf{Z}$. The main reason for treating mathematically only these cases is that we know of a simple way to write the associated spherically symmetric ansatz for the mapping of the q-dimensional spatial part of the probe world-volume onto the compact spherical target manifold. There obviously exist other spacetimes of interest to string and M-theory where brane probes could locally wrap more complicated compact manifolds that we here denote $\mathbf{X}_p$. The main criterion for that to be possible is that the homotopy mapping

$$\Pi_q(\mathbf{X}_p) \tag{5.1}$$

be non-trivial. To convince oneself that LWB excitations might be quite common, we refer the reader to ref. [27] where homotopy mappings to target spaces such as E_6, E_8 or G_2 are considered.

The excitation we described is a Dq- or Mq-brane that is wrapping the compact manifold $\mathbf{X}_p$ at one point of its world-volume. More concretely, the resulting configuration is composed of a fundamental Dq- or Mq-brane out of which another brane with p spatial dimensions protrudes at right angles. We also gave evidence that the wrapping might not be local if one is willing to take into account higher order curvature corrections to the brane probe action. It is conceivable that these will lead to a delocalization of the wrapping on a length scale which of the order of the fundamental scale of the theory, i.e., $\sqrt{\alpha'} = l_s$ or l_P, depending on whether one is considering branes in superstring or M-theory.

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The DBI action is a generalized $(q+1)$-dimensional volume which transpires in the expression for the energy of a LWB excitation eq. (2.19). Given the analysis performed for local wrappings based on the homotopy mapping $\Pi_q(\mathbf{S}^q)$, we can guess the generic form of the energy of an excitation associated with an arbitrary non-trivial mapping $\Pi_q(\mathbf{X}_p)$:

$$E = (\text{red shift}) T_q V_{\mathbf{X}_p} , \tag{5.2}$$

where T_q is the tension of the probe, $V_{\mathbf{X}_p}$ is the volume of the wrapped manifold calculated with a rescaled radius (see, e.g., eq. (2.8)), and the multiplicative red shift factor depends on the nature of the gravitational background probed.

In the supergravity examples considered, we have assumed that the brane probe was placed at a fixed radial distance ($\rho =$ const.) away from the horizon. We then used a spherically symmetric ansatz to find a profile function $F(r)$ that minimizes the energy of the configuration. It is quite possible that this procedure imposes too much constraint on the resulting brane system. In fact, it would seem natural to expect that the region where the brane wraps the compact manifold would move in the ρ direction so as to minimize its energy further. We therefore expect that there will be more structure to a LWB. This could be checked by relaxing the constraint $\partial_a X^{p+1} = \partial_a \rho = 0$ imposed in section 2. Doing so can be accomplished by introducing the function

$$h(r) = \rho(r) - \rho_0 , \tag{5.3}$$

where $\rho_0 =$ const. is the initial distance of the probe away from the horizon at $\rho = 0$. Then, one needs not only to minimize the energy of a LWB with respect to $F(r)$ but with respect to $h(r)$ with the boundary conditions $h(r \to +\infty) = 0$ as well. Recall that we have found the energy of a LWB with $h(r) = 0$ to be minimized when the probe is placed as close as possible to the horizon. We therefore expect that the energy of the brane excitation will be minimized for a negative $h(r)$. In other words, the resulting configuration will be a brane that locally wraps a compact manifold but is slightly deformed in the ρ direction toward the horizon.

The energy relation eq. (5.2) will also get modified when one takes into account curvature corrections to the much studied DBI action. Recall that the full D-brane action (with $B_{ab} = 0$, $F_{ab} = 0$) can be seen as an expansion in α', the first term being the DBI action. For example, we find that including the first order α' correction derived from bosonic string theory implies the following:

- The modification to the energy of the LWB is proportional to the curvature of the target space, i.e., the associated Ricci scalar. Consequently, having transverse manifolds with a large curvature compared to $1/\alpha'$ corresponds to a regime where the α' correction may not be negligible with respect to the $\mathcal{O}(\alpha'^2)$ corrections.

- If the system is in a regime where the curvature of the transverse space is small in such a way that the $\mathcal{O}(\alpha')$ correction is the dominant one, the LWB excitation is found to expand or delocalize itself to a size which is of the order of the string length.

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Consequently, the evidence we gave for a delocalization of the wrapping is not convincing when the curvature of the target space is large, but given it is small enough we find that there is definitely a regime for which the $\mathcal{O}(\alpha')$ correction dominates. For example, consider for simplicity the case where $H(\rho) = 1$ in eq. (2.7) with a spherical target space. Then, the curvature is proportional to $1/\rho^2$ where ρ is the radius of $\mathbf{S}^p$. The expansion of the D-brane action then schematically takes the form

$$S_{\text{D-brane}} \sim 1 + a_1 \frac{\alpha'}{\rho^2} + a_2 \frac{\alpha'^2}{\rho^4} + \cdots , \tag{5.4}$$

where the a_i's are dimensionless coefficients determined by string theory calculations. It is clear, given that the coefficients a_i are all of the same order of magnitude, that the validity of our conclusions regarding the delocalization of the soliton states is determined by whether ρ is large or not. For the D-branes in superstring theory, $a_1 = 0$ and the first curvature correction is the $\mathcal{O}(\alpha'^2)$ one. We have been unsuccessful at generalizing our numerical calculation to include this correction. The conclusion we can reach is that it is conceivable that LWB excitations in bosonic string theory will expand because of the curvature corrections although we have no way to predict whether taking into consideration higher order corrections will ultimately ruin this behavior. We can only take this bosonic calculation as an indication that a similar phenomenon might happen for the D-branes in superstring theory and the M-branes in M-theory.

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A. Atiyah-Manton solitons

In this appendix we present a scheme which allows one to obtain approximative solutions to the equations of motion associated with locally wrapped excitations and yet to easily deduce most of their relevant physical properties without resorting to a full numerical calculation. Recall that the contribution of the 'non-accelerating' part of the DBI action to the energy of a locally wrapped Dp-brane is[12]

$$E = T_p \int d^p\sigma \sqrt{g_\Omega} \left[(1 + \rho^2 F'^2)(r^2 + \rho^2 \sin^2 F)^{p-1} \right]^{1/2} - T_p \int d^p\sigma \sqrt{g_\Omega} r^{p-1} , \tag{A.1}$$

[12]For simplicity we consider here a non-physical background corresponding to setting $H(\rho) = 1$ in eq. (2.1) and with the dilaton Φ vanishing everywhere.

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where we have substracted the energy on an infinitely extended brane. We have introduced the notation $\sqrt{g_\Omega}$ to denote the volume factor associated with the angular variables on the world-volume.

It was proposed in ref. [26] that one can use the Atiyah-Manton ansatz for $F(r)$ in order to solve the differential equation associated with this profile. Let us first recall that the Atiyah-Manton ansatz originates from the computation of the holonomy of instanton solutions and as such, it should represent a three dimensional object. It is worth pointing out at this point that while, in principle, one could probably use the ansatz for other purposes, this is only justified for the $p = 3$ case. Applied to the excitations of interest, the Atiyah-Manton ansatz takes the form

$$F(r) = \omega\pi \left(1 - \frac{1}{\sqrt{1 + \frac{L^2}{r^2}}}\right), \qquad (A.2)$$

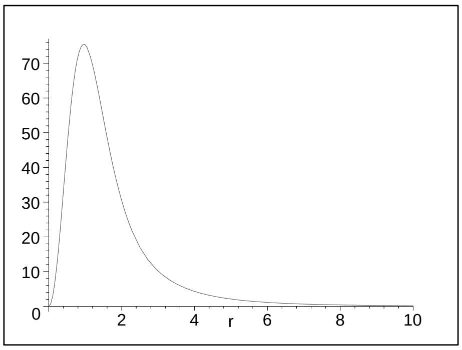

Figure 3: Energy density of an unstable locally wrapped D3-brane. The excitation has a typical world-volume size $L = 2\,l_s$ and the radius of the spherical co-cyle is $\rho = 4\,l_s$.

where ω is the winding number (associated with the topological charge) and L characterizes the size of the excitation, as will be made clear in a short while. We use this ansatz to reproduce the results of ref. [26], but with emphasis on the physical properties of the resulting soliton. We will later introduce a typical α' curvature correction and study its effects. Figure 3 shows the energy density as a function of the radial distance from the center of an unstable excitation for which the typical size is $L = 2\,l_s$. The radius of the spherical co-cycle on which the brane is wrapped was arbitrarily chosen to be $\rho = 4\,l_s$. In order to see why this configuration is unstable one needs to vary the energy functional with respect to the parameter L. As it turns out, the minimum of E is for $L = 0$ for all values of ρ. As explained earlier, one should not interpret the $L = 0$ states as being non-solitonic. One should see the $L \to 0$ as a limiting process in which one stays in the same topological sector (in this case $\Pi_3(S^3) = 1$).

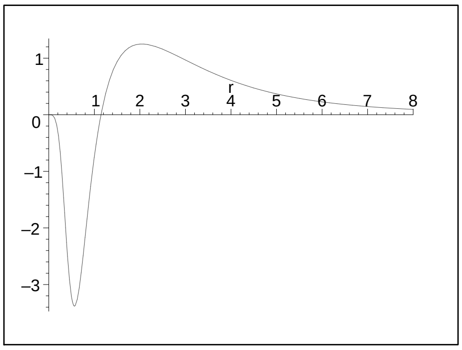

Figure 4: Energy density associated with the α' correction for $p = 3$, $\rho = l_s$ and $L = 10\,l_s$.

An interesting idea, proposed in ref. [26] in the context of brane-world scenarios, consists in investigating ways to stabilize the configuration to $L \neq 0$. We therefore extend the calculation by including the first order α' correction to the DBI action obtained from critical bosonic string theory [8]. The energy associated with this new term is found to be

$$E_{\alpha'} = \alpha' T_p(p-1) \int d^p\sigma \sqrt{g_\Omega} \left[(1+\rho^2 F'^2)(r^2+\rho^2 \sin^2 F)^{p-1}\right]^{1/2} \frac{MG' + (p-2)M'G(G-1)}{M^2 M' G^2}, \qquad (A.3)$$

where we have introduced, for simplicity,

$$M^2 = r^2 + \rho^2 \sin^2 F\,, \qquad G = \frac{1+\rho^2(F')^2}{M'^2}\,. \tag{A.4}$$

It is a generic feature, i.e., for all values of p, L and ρ, that this expression dips into negative values for $r \approx L$. This plays an important role in the delocalization of the wrapping. In fact, one can now plot the total energy, i.e., $E + E_{\alpha'}$, as a function of the delocalization parameter L. We find that there exists a minimum at finite L (of the order of the string lenght) for a wide range of the radial parameter ρ. It should be emphasized that the negative energy does not point to an instability of the solitonic configuration. One should see the system as being composed of an infinitely extended brane therefore possessing an infinite energy. We find that there will always be a minimum for finite L as long as the correction $E_{\alpha'}$ is large enough to maintain it. One can show that as the typical size of the target space $\mathbf{S}^p$ is increased, the term $E_{\alpha'}$ becomes increasingly small taking the minimum of the total energy to smaller and smaller values of L. Of course, when $E_{\alpha'}$ is large (L is then large as well), it is not a correction anymore and there is absolutely no reason why one should not include stringy corrections of order α'^2 and higher. There will nevertheless be a region where ρ (the radius of the target manifold) is small enough that $E_{\alpha'}$ is in fact a 'correction' to E that is significantly larger than the $\mathcal{O}(\alpha'^2)$ corrections. In this regime, the stabilization of the soliton to a finite world-volume size will not be affected by the $\mathcal{O}(\alpha'^2)$ terms.

The results of this appendix are, strictly speaking, only valid for locally wrapped D3-branes. As mentionned above one can hardly justify the use of the spherically symmetric Atiyah-Manton ansatz as a solution of the equations of motion for $p \neq 3$. With an appropriate ansatz one could probably generalize this simple approach to other D-branes.

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